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Optimized Compressive Strength Determination of Hybrid Nylon –Steel Fibre Reinforced Concrete [HNSFRC] Using Kings - Scheffe’s Binary Mixture

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ABSTRACT: Separate works done on nylon fibres and steel fibres by Nwachukwu and others (2022 d&f) and Nwachukwu and others (2022 b & g) respectively show that substitution of expensive conventional reinforcement with these fibres would result to increased compressive strength as well as low cost of concrete production. With this in mind and having known that it is an established fact that combining two or more types of fibers in a single concrete mixture would likely produce concrete properties that exhibit a synergetic response, such as increase in the compressive strength of the new composite, there is a likelihood that combination of nylon and steel fibres would produce a special concrete known as High Strength Fibre Reinforced Concrete [HSFRC]. Thus this research work is aimed at applying Scheffe’s Second Degree Model for six component mixtures [now known as Kings- Scheffe’s (6,2) model] to optimize the compressive strength of Hybrid Nylon - Steel Fibre Reinforced Concrete [HNSFRC]. It is important to note that Kings-Scheffe’s model is modified Scheffe’s model from six component and above while the term second degree is referred to binary mixtures, as third degree is referred to as ternary mixtures, apart from the individual mixtures at respective vertices. Using Scheffe’s (6,2) or [Kings- Scheffe’s (6,2)] simplex model introduced by Nwachukwu and others (2022h) , the compressive strengths of HNSFRC were obtained for different twenty –one mix proportions. The mix proportion of Nylon- Steel fibres was in 50% - 50% ratio for now. Twenty-one control experiments were also carried out, leading to the determination of the compressive strengths at the experimental control points. By using the Student’s t-test statistics, the adequacy of the model was validated .The 28th day optimum (maximum) compressive strength of HNSFRC was 101.25 MPa . This maximum value is higher than the minimum value specified by the American Concrete Institute (ACI), as 20 MPa as well as the minimum value specified by ASTM C 469, as 30.75 for good concrete. Thus, considering its safety, aesthetic and economic advantages, the HNSFRC compressive strength value can sustain construction of light-weight, heavy weight, commercial and industrial structures such as Bridge, Building pillars, Sidewalks, Building floors, Drainage pipes, Septic tanks, Concrete Flooring for parking lots, Playgrounds, Airport runways, Taxiways, Maintenance hangars, Access roads, Workshops, Port pavements, Container storage and Handling areas, Bulk storage warehouses, and Military warehouses as high strength and high performance concrete.

KEYWORDS: HNSFRC, Kings- Scheffe’s (6,2) Optimization Model, Compressive Strength, Mixture Design

I. INTRODUCTION

Concrete which is classified as the most widely used construction material has been undergoing changes both as a material and due to technological advancement. In an attempt to solve the world’s housing deficit due to increasing cost of construction materials especially the conventional reinforcement, there have been several research outputs, one being the partial or wholly replacement of the expensive conventional reinforcement with fibres in the concrete production. By definition, concrete is defined by Oyenuga (2008) as a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. As stated by Syal and Goel, (2007), concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world. The key role being played by concrete is emphasised by Neville (1990) where he acknowledged that concrete plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. However, Shetty (2006) highlighted some of drawbacks associated with concrete, especially the plain type. According to him, plain concrete

possesses a very low tensile strength, limited ductility and little resistance to cracking. As a way of finding solution to this situation, there have been continuous search for the upgrading of the concrete properties. In line with this, attempts have been made in the past to improve the tensile properties of concrete members by way of using conventional reinforced steel bars. Despite the fact that both these methods provide tensile strength to the concrete members, they however, do not increase the inherent tensile strength of concrete itself. Subsequent upon further researches and recent developments in concrete technology, it has been established that the addition of fibres to concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This type of concrete is known as Fibre reinforced concrete [FRC]. Hybrid Fibre Reinforced Concrete [HFRC] is the use of two or more fibres in a single concrete mixture matrix with the aim of improving its overall properties. Hybrid Nylon- Steel Fibre Reinforced Concrete [HNSFRC] is concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with the combination of nylon fibre and steel fibre. The special property of concrete to be investigated is the concrete's compressive strength. Compressive strength of concrete is the strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine [UTM]. Further, the compressive strength of the concrete cube test also provides an idea about all the characteristics of concrete under examination.

The two fibres in this work can be combined with other components through the process of optimization. In general, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated limitations or boundaries placed on the concerned variables. Specifically, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. According to Shacklock (1974), one of the objectives of mix design is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work. Another classic definition by Jackson and Dhir (1996) sees concrete mix design as the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. In line with the above definition, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) appeared to be more complex and time consuming as they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Therefore, optimization of the concrete mixture design remains the fastest method, best option, most convenient and the most efficient way of selecting concrete mix proportions for better efficiency and better performance of concrete when compared with usual empirical methods listed above. An example of optimization model is Scheffe's Model. However, Scheffe's Model could only reflect up to four component mixture, with little expansion to five component mixtures. But through modification of the same Scheffe's model, the works of Nwachukwu and others (2022h, 2022i, 2022j, 2023a, 2023b, 2023c, 2023e, 2024a, 2024b, and 2024d) have expanded the use of this Scheffe's model to six component mixtures and still looking forward to publishing works on seven component mixtures. Thus, the modified Scheffe's model in six or seven or higher component mixtures is better described as Kings- Scheffe's Optimization Model. It could be in the form of Scheffe's Second Degree Model [in binary mixtures- that A_{12} , A_{23} , A_{45} , etc.] or Scheffe's Third Degree Model [in ternary mixtures- that is A_{112} , A_{233} , A_{245} , etc]. Thus, in this present study, Kings- Scheffe's Second Degree Model for six components mixtures (namely, water, cement, fine aggregate, coarse aggregate, nylon fibre and steel fibre) will be in focus.

This present study examines the application of Kings-Scheffe's Second Degree Model for six component mixture in the optimization of the compressive strength of HNSFRC. Despite the fact that some related works have been done by many researchers, none has been able to address the real subject matter. For instance, on NFRC and other related works, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete. Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively. Hassan and others (2022) investigated the Mechanical Properties and Absorption of High-Strength Fiber-Reinforced Concrete (HSFRC) with Sustainable Natural Fibers. On SFRC and other related works, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated

temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate. Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again, Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete. Yew and others (2011) have investigated the strength properties of Hybrid Nylon-Steel fibre-reinforced concrete in comparison to that of polypropylene-steel fibre-reinforced concrete. Recent works on optimization show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For instance, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were based on the use of Scheffe' model in the optimization of compressive strength of Periwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Mbadike and Osadebe (2013) applied Scheffe's (4,2) model to optimize the compressive strength of Laterite Concrete. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRF). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRF where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation, Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC). Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model InThe Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and Opara (2023) in their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)- Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength

of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Nwachukwu and others (2024c) applied Scheffe’s (5,2) model to evaluate the compressive strength of Plastic Fibre Reinforced Concrete [PLFRC]. Nwachukwu and others (2024d) applied the use of Scheffe’s Third Degree Model to optimize the compressive strength of HPNFRC. Nwachukwu and others (2024e) applied the use of Scheffe’s Third Degree Regression Model to optimize the compressive strength of MSFRC. Nwachukwu and others (2024f) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of NFRC. Again, Nwachukwu and others (2024g) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Nwachukwu and others (2024h) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Finally, Nwachukwu and others (2024i) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of SFRC. Facts from the works reviewed so far show that no work has been done on the use of Kings- Scheffe’s Second Degree Model [or Kings- Scheffe’s binary mixture] to optimize the compressive strength of HNSFRC. Thus, there is urgent need for this present research work.

II. METHODOLOGY

2.1 MATERIALS FOR HNSFRC- CS MIXTURES

In this research work, the HNSFRC component materials under Compressive Strength [CS] examination in line with Kings- Scheffe’s (6,2) model are Water/Cement ratio, Cement, Fine and Coarse Aggregates, Nylon and Steel Fibres . The water is obtained from potable water from the clean water source and was used in accordance with ASTM C1602/C1602M-22 (2022). The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). Fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. Both fine and coarse aggregates are procured and prepared in accordance with ASTM C33/C33M-18 (2018). The same size and nature of nylon fibre and steel fibre used previously by Nwachukwu and others (2022 d & f) and Nwachukwu and others (2022 b & g) respectively, are the same as the one being used in this present work.

2.2. GENERAL BACKGROUND INFORMATION ON SCHEFFE’S / KINGS –SCHEFFE’S (6,2) MODEL

The difference between the Kings- Scheffe’s model and the original Scheffe’s model is that Kings –Scheffe’s model is the upgraded/ modified/ expanded Scheffe’s model to accommodate from six component mixtures whether second degree [Binary] or third degree[Ternary] form. Thus, many of the technical terms used in the original Scheffe’s model are also used here. Therefore, both models are used here interchangeably for this six component mixture. As usual, a simplex lattice is a structural representation of lines joining the atoms of a particular mixture. Expectedly, these atoms are constituent components of that same mixture. For this present HNSFRC mixture, the constituent elements are the following six components: water, cement, fine aggregate, coarse aggregate, nylon fibre and steel fibre. This shows that a simplex of six-component mixture is a five -dimensional solid. Mixture components, according to Obam (2009) are subject to the constraint that the sum of all the components must be equal to 1. That is: $X_1 + X_2 + X_3 + \dots + X_q = 1$; $\Rightarrow \sum_{i=1}^q X_i = 1$ (1)

where $X_i \geq 0$ and $i = 1, 2, 3 \dots q$, and q = the number of mixtures.

2.2.1. POSSIBLE DESIGN POINTS FOR HNSFRC KINGS- SCHEFFE’S (6, 2) MIXTURES

The Scheffe’s (q, m) simplex lattice design is characterized by the symmetric arrangements of points within the experimental region and a well-chosen regression equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1$; $i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe’s polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^4C_2 = 6$ points. Each X_i can take $m+1 = 3$ possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{2}, \frac{1}{2}, 0), (0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2})$. To evaluate the number of coefficients/terms/ design points required for a given lattice , the following general formula is employed:

$$k = \frac{(q+m-1)!}{(q-1)! \cdot m!} \quad \text{Or} \quad {}^{q+m-1}C_m \quad \mathbf{2(a-b)}$$

Where k = number of coefficients/ terms / design points , q = number of components/mixtures = 6 in this present study and m = number of degree of polynomial = 2 in this present work. Using either of Eqn. (2), $k_{(6,2)} = 21$. This implies that the possible design points for Scheffe’s (6,2) lattice can be as follows:

$$A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0); A_5 (0,0,0,0,1,0); A_6 (0,0,0,0,0,1); A_{12} (0.67,0.33,0,0,0,0); A_{13} (0.67,0,0.33,0,0,0); A_{14} (0.67,0,0,0.33,0,0); A_{15} (0.67,0,0,0,0.33,0); A_{16} (0.67,0,0,0,0,0.33); A_{23} (0,0.50,0.50,0,0,0); A_{24} (0,0.50,0,0.50,0,0); A_{25} (0,0.50,0,0,0.50,0); A_{26} (0,0.50,0,0,0.50,0); A_{34} (0.50,0.50,0,0,0,0); A_{35} (0.50,0,0.50,0,0,0); A_{36} (0.50,0,0,0.50,0,0); A_{45} (0.50,0,0,0,0.50,0); A_{46} (0.50,0,0,0,0.50,0); A_{56} (0,0,0.50,0.50,0,0); \tag{3}$$

Again according to Obam (2009), a Scheffe’s polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \dots X_q$ is given in the form of Eqn.(4) under.

$$N = b_0 + \sum b_i x_i + \sum b_{ij} x_j + \sum b_{ijk} x_j x_k + \dots + \sum b_{i_1 i_2 \dots i_n} x_{i_1} x_{i_2} \dots x_{i_n} \tag{4}$$

where ($1 \leq i \leq q, 1 \leq i \leq j \leq k \leq q, 1 \leq i_1 \leq i_2 \leq \dots \leq i_n \leq q$ respectively) , b = constant coefficients and N is the response which represents the property under investigation, which ,in this case is the compressive strength.

As this research work is based on the Scheffe’s (6, 2) simplex, the actual form of Eqn. (4) for six component mixture , degree two has been developed by Nwachukwu and others (2022h) and will be applied subsequently in this present work.

2.2.2. PSEUDO AND ACTUAL COMPONENTS IN HNSFRC SCHEFFE’S MIX DESIGN

In Scheffe’s mix design, the relationship between the actual components and the pseudo components has been established as $Z = A * X$ (5)

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship Re-arranging Eqn. (5) yields: $X = A^{-1} * Z$ (6)

2.2.3. HNSFRC OPTIMIZATION EQUATION FOR KINGS- SCHEFFE’S (6,2) LATTICE

The polynomial equation by Scheffe (1958), which is also known as response is given in Eqn.(4). But Eqn.(4) has been developed by Nwachukwu and others (2022h) to accommodate six component mixture for Scheffe’s second degree model .Hence, the formulated polynomial equation for Scheffe’s (6,2) simplex lattice based on Eqn.(4) is shown in Eqn.(7):

$$N = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{16} X_1 X_6 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{26} X_2 X_6 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{36} X_3 X_6 + \beta_{45} X_4 X_5 + \beta_{46} X_4 X_6 + \beta_{56} X_5 X_6 \tag{7}$$

2.2.4. COEFFICIENTS DETERMINATION OF THE HNSFRC–CS - SCHEFFE’S (6,2) POLYNOMIAL

From the work of Nwachukwu and others (2022h), the coefficients of the Scheffe’s (6, 2) polynomial are expressed as under. :

$$\begin{aligned} \beta_1 &= N_1; \beta_2 = N_2; \beta_3 = N_3; \beta_4 = N_4; \beta_5 = N_5 \text{ and } \beta_6 = N_6 & \mathbf{8(a-f)} \\ \beta_{12} &= 4N_{12} - 2N_1 - 2N_2; \beta_{13} = 4N_{13} - 2N_1 - 2N_3; \beta_{14} = 4N_{14} - 2N_1 - 2N_4; & \mathbf{9(a-c)} \\ \beta_{15} &= 4N_{15} - 2N_1 - 2N_5; \beta_{16} = 4N_{16} - 2N_1 - 2N_6; \beta_{23} = 4N_{23} - 2N_2 - 2N_3; \beta_{24} = 4N_{24} - 2N_2 - 2N_4; & \mathbf{10(a-d)} \\ \beta_{25} &= 4N_{25} - 2N_2 - 2N_5; \beta_{26} = 4N_{26} - 2N_2 - 2N_6; \beta_{34} = 4N_{34} - 2N_3 - 2N_4; \beta_{35} = 4N_{35} - 2N_3 - 2N_5; & \mathbf{11(a-d)} \\ \beta_{36} &= 4N_{36} - 2N_3 - 2N_6; \beta_{45} = 4N_{45} - 2N_4 - 2N_5; \beta_{46} = 4N_{46} - 2N_4 - 2N_6; \beta_{56} = 4N_{56} - 2N_5 - 2N_6; & \mathbf{12(a-d)} \end{aligned}$$

Where N_i = Response Function (or Compressive Strength) for the pure component, i

2.2.5. HNSFRC SCHEFFE’S (6, 2) MIXTURE DESIGN MODEL

By substituting Eqns. (8)-(12) into Eqn. (7), yields the mixture design model for the HNSFRC Scheffe’s (6,2) lattice.

2.2.6. EVALUATING PSEUDO AND ACTUAL MIX RATIOS FOR THE HNSFRC KINGS- SCHEFFE’S (6, 2) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINTS [IETP] AND EXPERIMENTAL CONTROL TEST POINTS [ECTP].

A. AT THE HNSFRC IETP

Normally, the conventional mix ratio is usually in the form of 1:2:4. However this conventional nomenclature is impossible to actualize because of the requirement of simplex lattice design based on Eqn. (1) criteria at a given water/cement ratio for the actual mix ratio. Thus, there is need for the transformation of the actual components proportions to meet the Eqn. (1) . Based on experience and knowledge from similar six component degree two work where the fibres mix ratios are at 50: 50 as well as previous knowledge from literature, the following arbitrary prescribed mix ratios are chosen for the six vertices of Scheffe’s (6,2) lattice. They are as follows :

$$A_1 (0.67:1:1.7:2:0.5:0.5); A_2 (0.56:1:1.6:1.8:0.8:0.8); A_3 (0.5:1:1.2:1.7:1:1); A_4 (0.7:1:1:1.8:1.2:1.2); A_5 (0.75:1:1.3:1.2:1.5:1.5), \text{ and } A_6 (0.80:1:1.3:1.2:0.9:0.9) \tag{13}$$

which represent water/cement ratio, cement, fine aggregate, coarse aggregate, nylon fibre and steel fibre respectively. For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures are always chosen:

$$A_1(1:0:0:0:0:0), A_2(0:1:0:0:0:0), A_3(0:0:1:0:0:0), A_4(0:0:0:1:0:0), A_5(0:0:0:0:1:0) \text{ and } A_6(0:0:0:0:0:1) \tag{14}$$

The rest are shown in Eqn.(3). For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. By substituting the mix ratios from point A₁ into Eqn. (5), we obtain:

$$\begin{pmatrix} 0.67 \\ 1.00 \\ 1.70 \\ 2.00 \\ 0.50 \\ 0.50 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

Transforming the R.H.S matrix and solving, we obtain as follows:

A₁₁(1) + A₂₁(0) + A₃₁(0) + A₄₁(0) + A₅₁(0) + A₆₁(0) = 0.67. Thus, A₁₁ = 0.67. Similarly, A₂₁= 1; A₃₁= 1.7; A₄₁= 2; A₅₁= 0.5; A₆₁= 0.5. The same approach is used to obtain the remaining values as shown in Eqn. (16)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} \begin{pmatrix} 1 X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} \tag{16}$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(16) will yield the corresponding actual mix ratios as follows: For instance, considering point A₁₂ we have: A₁₂ (0.67, 0.33, 0, 0, 0, 0). Thus,

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \\ 1.60 \end{pmatrix} \tag{17}$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$; $Z_4 = 1.90$; $Z_5 = 1.60$ and $Z_6 = 1.60$

The same approach goes for the remaining mid-point mix ratios. Hence, in order to generate the 21 polynomial coefficients, twenty-one (21) experimental tests will be carried out and the corresponding mix ratios are as depicted in Table 1.

Table 1: Pseudo (X) And Actual (Z) Mix Ratio For HNSFRC- CS Based On Kings-Scheffe’s (6,2) Lattice For IETP

S/N	IETP	PSEUDO COMPONENT						RESPONSE SYMBOL	ACTUAL COMPONENT					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆
1	E ₁	1	0	0	0	0	0	N ₁	0.67	1.00	1.70	2.0	0.5	0.5
2	E ₂	0	1	0	0	0	0	N ₂	0.56	1.00	1.60	1.8	0.8	0.8
3	E ₃	0	0	1	0	0	0	N ₃	0.50	1.00	1.20	1.7	1.0	1.0
4	E ₄	0	0	0	1	0	0	N ₄	0.70	1.00	1.00	1.8	1.2	1.2
5	E ₅	0	0	0	0	1	0	N ₅	0.75	1.00	1.30	1.2	1.5	1.5
6	E ₆	0	0	0	0	0	1	N ₆	0.63	1.00	1.67	1.9	1.6	1.6
7	E ₁₂	0.67	0.33	0	0	0	0	N ₁₂	0.60	1.00	1.63	1.8	0.7	0.7
8	E ₁₃	0.67	0	0.33	0	0	0	N ₁₃	0.61	1.00	1.54	1.9	0.6	0.6
9	E ₁₄	0.67	0	0	0.33	0	0	N ₁₄	0.56	1.00	1.37	1.8	0.8	0.8
10	E ₁₅	0.67	0	0	0	0.33	0	N ₁₅	0.68	1.00	1.47	1.9	0.7	0.7
11	E ₁₆	0.67	0	0	0	0	0.33	N ₁₆	0.69	1.00	1.23	1.8	0.9	0.9
12	E ₂₃	0	0.50	0.50	0	0	0	N ₂₃	0.70	1.00	1.57	1.7	0.8	0.8
13	E ₂₄	0	0.50	0	0.50	0	0	N ₂₄	0.72	1.00	1.43	1.4	1.1	1.1
14	E ₂₅	0	0.50	0	0	0.50	0	N ₂₅	0.55	1.00	1.40	1.7	0.8	0.8
15	E ₂₆	0	0.50	0	0	0	0.50	N ₂₆	0.52	1.00	1.20	1.7	0.9	0.9
16	E ₃₄	0.50	0.50	0	0	0	0	N ₃₄	0.61	1.00	1.67	1.8	0.9	0.9
17	E ₃₅	0.50	0	0.50	0	0	0	N ₃₅	0.66	1.00	1.73	1.8	1.0	1.0
18	E ₃₆	0.50	0	0	0.50	0	0	N ₃₆	0.63	1.00	1.50	1.6	0.7	0.7
19	E ₄₅	0.50	0	0	0	0.50	0	N ₄₅	0.69	1.00	1.40	1.4	0.6	0.6
20	E ₄₆	0.50	0	0	0	0	0.50	N ₄₆	0.57	1.00	1.13	1.7	1.0	1.0
21	E ₅₆	0	0	0.50	0.50	0	0	N ₅₆	0.64	1.00	1.07	1.7	1.1	1.1

B. AT THE HNSFRC ECTP

Here, twenty- one (21) different controls were predicted which according to Scheffe’s (1958) ,their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2: Actual and Pseudo Component of HNSFRC Based on Kings-Scheffe (6,2) Lattice for ECTP

S/N	ECTP	PSEUDO COMPONENT						RESPONSE SYMBOL	ACTUAL COMPONENT					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆
1	C ₁	0.25	0.25	0.25	0.25	0	0	N ₁	0.61	1	1.38	1.83	0.5	0.50
2	C ₂	0.25	0.25	0.25	0	0.25	0	N ₂	0.62	1	1.45	1.68	0.8	0.8
3	C ₃	0.25	0.25	0	0.25	0.25	0	N ₃	0.67	1	1.40	1.70	1	1
4	C ₄	0.25	0	0.25	0.25	0.25	0	N ₄	0.66	1	1.30	1.68	1.2	1.2
5	C ₅	0	0.25	0.25	0.25	0.25	0	N ₅	0.63	1	1.28	1.63	1.5	1.5
6	C ₆	0.20	0.20	0.20	0.20	0.20	0	N ₆	0.64	1	1.36	1.70	0.65	0.65
7	C ₁₂	0.30	0.30	0.30	0.10	0	0	N ₁₂	0.59	1	1.45	1.83	0.75	0.75
8	C ₁₃	0.30	0.30	0.30	0	0.10	0	N ₁₃	0.59	1	1.48	1.77	0.85	0.85
9	C ₁₄	0.30	0.30	0	0.30	0.10	0	N ₁₄	0.65	1	1.42	1.80	1	1
10	C ₁₅	0.30	0	0.30	0.30	0.10	0	N ₁₅	0.64	1	1.30	1.77	0.9	0.9
11	C ₁₆	0	0.30	0.30	0.30	0.10	0	N ₁₆	0.60	1	1.27	1.71	1	1
12	C ₂₃	0.10	0.30	0.30	0.30	0	0	N ₂₃	0.60	1	1.31	1.79	1.55	1.55
13	C ₂₄	0.30	0.10	0.30	0.30	0	0	N ₂₄	0.62	1	1.33	1.83	1.1	1.1
14	C ₂₅	0.30	0.10	0.30	0.30	0	0	N ₂₅	0.63	1	1.41	1.85	1.25	1.25
15	C ₂₆	0.10	0.20	0.30	0.40	0	0	N ₂₆	0.61	1	1.25	1.79	1.35	1.35
16	C ₃₄	0.30	0.20	0.10	0.40	0	0	N ₃₄	0.64	1	1.35	1.85	0.89	0.89
17	C ₃₅	0.20	0.20	0.10	0.10	0.40	0	N ₃₅	1.40	1	1.04	1.59	1.08	1.08
18	C ₃₆	0.30	0.10	0.30	0.20	0.10	0	N ₃₆	0.62	1	1.36	1.77	0.92	0.92
19	C ₄₅	0.25	0.25	0.15	0.15	0.20	0	N ₄₅	0.61	1	1.51	3.16	0.91	0.91
20	C ₄₆	0.30	0.30	0.20	0.10	0.10	0	N ₄₆	0.68	1	1.56	1.96	0.98	0.98
21	C ₅₆	0.10	0.30	0.30	0.30	0	0	N ₅₆	1.30	1	1.31	1.79	0.95	0.95

2.2.7. MEASUREMENT OF QUANTITIES OF HNSFRC- CS MATERIALS

The actual component as transformed from Eqn. (17) , Table (1) and (2) were used to measure out the quantities of water/cement ratio (Z₁), cement (Z₂), fine aggregate (Z₃), coarse aggregate (Z₄), nylon fibre (Z₅) and steel fibre (Z₆) in their respective ratios using a weighing balance of 50kg capacity for the eventual Concrete Cube strength test at the laboratory.

Mathematically, Measured Quantity, M^Q of HNSFRC Mixture is given by Eqn.(18)

$$M^Q = \frac{X}{T} * Y \tag{18}$$

Where, X = Individual mix ratio at each test point. For example, X = 0.67 for Z₁ at E₁ in Table 1.

T = Sum of mix ratios at each test point = 5.37 at E₁ in Table 1.

And Y = Average weight of Concrete cube/beam/cylinder

For the Compressive Strength concrete cube mould of 15cm*15cm*15cm, Average Y from experience = 8kg
 Samples of measured quantities can be seen from the works of Nwachukwu and others 2024 (a and b).

2.3. METHODS FOR HNSFRC COMPRESSIVE STRENGTH [CS] TEST

2.3.1. HNSFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR COMPRESSIVE STRENGTH TEST

The specimen used for the compressive strength is concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. As usual, the mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. For the twenty one experimental tests, a total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty one, out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8)

– (12). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28 days of curing the specimens were taken out of the curing tank for the compressive strength test.

2.3.2. HNSFRC COMPRESSIVE STRENGTH TEST PROCEDURE/CALCULATION

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. Two samples were crushed for each mix ratio and in each case, the compressive strength was calculated using Eqn.(18)

$$\text{Compressive Strength} = \frac{\text{Average failure Load, P (N)}}{\text{Cross-sectional Area, A (mm}^2\text{)}} \tag{19}$$

III. RESULTS PRESENTATION AND DISCUSSION

3.1. HNSFRC RESPONSES FOR THE IETP.

The results of the compressive strength (R_{response}, N_i) of HNSFRC based on a 28-days strength is presented in Table 3. These are calculated from Eqn..(18)

Table 3: 28th Day Compressive Strength (Responses) for HNSFRC Based on Kings-Scheffe’s (6, 2) Model for the IETP.

S/N	IETP	EXPERIMENTAL NUMBER	RESPONSE N_i , MPa	RESPONSE SYMBOL	$\sum N_i$	AVERAGE RESPONSE N, MPa
1	E ₁	HNSFRC/CS/ E ₁ A	66.78	N ₁	133.99	67.00
		HNSFRC/CS/ E ₁ B	67.21			
2	E ₂	HNSFRC/CS/ E ₂ A	79.42	N ₂	159.54	79.77
		HNSFRC/CS/ E ₂ B	80.12			
3	E ₃	HNSFRC/CS/ E ₃ A	87.88	N ₃	173.10	86.55
		HNSFRC/CS/ E ₃ B	85.22			
4	E ₄	HNSFRC/ CS/E ₄ A	96.54	N ₄	191.86	95.93
		HNSFRC/CS/ E ₄ B	95.32			
5	E ₅	HNSFRC/CS/ E ₅ A	84.43	N ₅	169.66	84.83
		HNSFRC/CS/ E ₅ B	85.23			
6	E ₆	HNSFRC/CS/ E ₆ A	101.27	N ₆	202.50	101.25
		HNSFRC/CS/ E ₆ B	101.23			
7	E ₁₂	HNSFRC/CS/ E ₁₂ A	86.24	N ₁₂	172.56	86.28
		HNSFRC/CS/ E ₁₂ B	86.32			
8	E ₁₃	HNSFRC/CS/ E ₁₃ A	76.65	N ₁₃	154.87	77.44
		HNSFRC/CS/ E ₁₃ B	78.22			
9	E ₁₄	HNSFRC/CS/ E ₁₄ A	64.32	N ₁₄	129.55	64.78
		HNSFRC/CS/ E ₁₄ B	65.23			
10	E ₁₅	HNSFRC/ CS/E ₁₅ A	56.21	N ₁₅	114.53	57.27
		HNSFRC/CS/ E ₁₅ B	58.32			

11	E ₁₆	HNSFRC/CS/ E ₁₆ A HNSFRC/CS/ E ₁₆ B	85.43 86.37	N ₁₆	171.80	85.90
12	E ₂₃	HNSFRC/CS/ E ₂₃ A HNSFRC/CS/ E ₂₃ B	98.32 97.13	N ₂₃	195.64	97.82
13	E ₂₄	HNSFRC/CS/ E ₂₄ A HNSFRC/CS/ E ₂₄ B	71.24 72.21	N ₂₄	143.45	71.73
14	E ₂₅	HNSFRC/ CS/E ₂₅ A HNSFRC/CS/ E ₂₅ B	56.39 58.18	N ₂₅	114.57	57.29
15	E ₂₆	HNSFRC/CS/ E ₂₆ A HNSFRC/CS/ E ₂₆ B	48.85 48.36	N ₂₆	97.21	48.61
16	E ₃₄	HNSFRC/ CS/E ₃₄ A HNSFRC/CS/ E ₃₄ B	42.20 42.16	N ₃₄	84.36	42.18
17	E ₃₅	HNSFRC/ CS/E ₃₅ A HNSFRC/CS/ E ₃₅ B	75.23 76.21	N ₃₅	151.44	75.72
18	E ₃₆	HNSFRC/CS/ E ₃₆ A HNSFRC/CS/ E ₃₆ B	89.43 90.11	N ₃₆	179.54	89.77
19	E ₄₅	HNSFRC/ CS/E ₄₅ A HNSFRC/CS/ E ₄₅ B	60.21 60.18	N ₄₅	120.39	60.20
20	E ₄₆	HNSFRC/ CS/E ₄₆ A HNSFRC/ CS/E ₄₆ B	78.21 79.32	N ₄₆	157.53	78.77
21	E ₅₆	HNSFRC/ CS/E ₅₆ A HNSFRC/ CS/E ₅₆ B	56.32 57.19	N ₅₆	113.51	56.76

3.2. HNSFRC RESPONSES FOR THE ECTP.

Table 4 shows the 28th day Compressive strength results for the ECTP.

Table 4: 28TH Day Compressive Strength (Responses) Results for HNSFRC Based on Kings-Scheffe’s (6,2) Model for the ECTP .

S/N	ECTP	EXPERIMENTAL NUMBER	RESPONSE, MPa	AVERAGE RESPONSE, MPa
1	C ₁	HNSFRC/CS/ C ₁ A HNSFRC/CS/ C ₁ B	66.71 67.24	66.98
2	C ₂	HNSFRC/CS/ C ₂ A HNSFRC/CS/ C ₂ B	79.32 80.76	80.04
3	C ₃	HNSFRC/CS/ C ₃ A HNSFRC/CS/ C ₃ B	87.76 85.45	86.61
4	C ₄	HNSFRC/ CS/C ₄ A HNSFRC/CS/ C ₄ B	96.47 95.43	95.95

5	C ₅	HNSFRC/CS/ C ₅ A HNSFRC/CS/ C ₅ B	84.54 85.67	85.11
6	C ₆	HNSFRC/CS/ C ₆ A HNSFRC/CS/ C ₆ B	100.00 97.45	98.73
7	C ₁₂	HNSFRC/CS/ C ₁₂ A HNSFRC/CS/ C ₁₂ B	86.43 85.21	85.82
8	C ₁₃	HNSFRC/CS/ C ₁₃ A HNSFRC/CS/ C ₁₃ B	76.43 74.32	75.38
9	C ₁₄	HNSFRC/CS/ C ₁₄ A HNSFRC/CS/ C ₁₄ B	64.21 68.23	66.22
10	C ₁₅	HNSFRC/ CS/C ₁₅ A HNSFRC/CS/ C ₁₅ B	56.03 58.34	57.19
11	C ₁₆	HNSFRC/CS/ C ₁₆ A HNSFRC/CS/ C ₁₆ B	85.12 86.21	85.67
12	C ₂₃	HNSFRC/CS/ C ₂₃ A HNSFRC/CS/ C ₂₃ B	98.31 97.21	97.76
13	C ₂₄	HNSFRC/CS/ C ₂₄ A HNSFRC/CS/ C ₂₄ B	71.23 73.22	72.23
14	C ₂₅	HNSFRC/ CS/C ₂₅ A HNSFRC/CS/ C ₂₅ B	56.10 57.21	56.66
15	C ₂₆	HNSFRC/CS/ C ₂₆ A HNSFRC/CS/ C ₂₆ B	47.23 45.34	46.29
16	C ₃₄	HNSFRC/ CS/C ₃₄ A HNSFRC/CS/ C ₃₄ B	42.56 42.45	42.51
17	C ₃₅	HNSFRC/ CS/C ₃₅ A HNSFRC/CS/ C ₃₅ B	73.64 72.35	73.00
18	C ₃₆	HNSFRC/CS/ C ₃₆ A HNSFRC/CS/ C ₃₆ B	89.21 90.35	89.78
19	C ₄₅	HNSFRC/ CS/C ₄₅ A HNSFRC/CS/ C ₄₅ B	60.12 62.11	61.12
20	C ₄₆	HNSFRC/ CS/C ₄₆ A HNSFRC/ CS/C ₄₆ B	78.10 77.21	77.66
21	C ₅₆	HNSFRC/ CS/C ₅₆ A HNSFRC/ CS/C ₅₆ B	56.23 57.00	56.62

3.3. KINGS-SCHEFFE'S (6,2) MODEL FOR THE HNSFRC RESPONSES

By substituting the values of the compressive strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients ($\beta_1, \beta_2 \dots \beta_{34}, \beta_{35} \dots \beta_{56}$) of the Kings- Scheffe's second degree polynomial for HNSFRC. Substituting the values of these coefficients into Eqn. (7) yields the polynomial model for the optimization of the compressive strengths of HNSFRC (at 28th day) based on Kings-Scheffe's (6,2) lattice.

3.4. SCHEFFE'S (6,2) MODEL RESPONSES FOR HNSFRC AT ECTP

By substituting the pseudo mix ratio of points $C_1, C_2, C_3, C_4, C_5, \dots C_{56}$ of Table 4 into revised Eqn.(7), we obtain the Scheffe's second degree model responses for the ECTP of HNSFRC.

3.5. TEST OF ADEQUACY OF THE KINGS-SCHEFFE'S (6,2) MODEL FOR HNSFRC

In this session, the test of adequacy is performed to check the correlation between the compressive strength results (lab responses) given in Table 4 and model responses from the ECTP as obtained in Session 4.4 using the Student's – T - test. The procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The result of the test shows that there is no significant difference between the experimental results and model responses. Therefore, the model is very adequate for predicting the compressive strength of HNSFRC based on Scheffe's (6,2) lattice.

3.6. RESULTS DISCUSSION

The Optimum (maximum) compressive strength of HNSFRC based on Scheffe's (6,2) lattice is **101.25MPa** . This corresponds to mix ratio of **0.63:1.00: 1.67:1.9:1.6:1.6** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Nylon Fibre and Steel Fibre respectively. Similarly, the optimum (minimum) compressive strength is **42.18MPa** which also correspond to the mix ratio of **0.61: 1.00:1.67:1.8:0.9:0.9** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Nylon Fibre and Steel Fibre respectively. The maximum value from the model was found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete and also minimum standard (of 4500psi or 30.75MPa) specified by the American Society of Testing and Machine, ASTM C 39 and ASTM C 469. Thus, the model can be used to obtain the HNSFRC compressive strength of all points (1 - 56) in the simplex based on Scheffe's Second Degree Model.

IV. CONCLUSION

So far in this research work, Kings- Scheffe's Second Degree Optimization Model for HNSFRC has been presented. The Kings- Scheffe's Method was used to predict the mix ratios as well as a model for predicting the compressive strength of HNSFRC. By using Kings-Scheffe's (6,2) simplex model, the values of the compressive strength were obtained at all 21 points(1- 56). The result of the student's t-test confirmed that there is a good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable compressive strength predicted by the model based on Scheffe's (6,2) model is as recorded in the result discussion session. As expected, the maximum value meets the minimum standard requirement (of 20 MPa and 30.75MPa) stipulated by American Concrete Institute (ACI) and American Society of Testing and Machine, ASTM C 39 and ASTM C 469 respectively, for the compressive strength of good and high performance concrete. Thus, with the Kings- Scheffe's (6,2) model, any desired strength, given any mix proportions can be easily predicted and evaluated and vice versa.

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