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Flexural and Split Tensile Strengths Evaluation of Hybrid Nylon- Steel Fibre Reinforced Concrete [HNSFRC] Based on Kings - Scheffe's (6,2) Optimization Model.

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ABSTRACT: By definition, flexural strength is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. Sometimes, when two or more fibres are combined and added to concrete in various mixture designs as replacement for conventional reinforcement, then superior reinforced concrete is produced with promising mechanical properties such as flexural strength that can control cracking due to plastic shrinkage and to drying shrinkage. This research work is aimed at using Kings- Scheffe's Second Degree Model for six component mixture to optimize the Flexural Strength and Split Tensile Strength of Hybrid Nylon - Steel Fibre Reinforced Concrete [HNSFRC]. It is important to note that Kings- Scheffe's model is modified Scheffe's model from six components and above Using Kings- Scheffe's Simplex method, the Flexural Split Tensile Strengths [FSTS] of HNSFRC were determined for different mix proportions. Control experiments were also carried out and the flexural and split tensile strengths determined. The test statistics using the Student's t-test validated the results. Maximum design strengths recorded for the flexural test at 14 and 28 days were 6.75MPa and 10.39MPa respectively, while those recorded for the splitting tensile test were 4.46MPa and 6.80MPa respectively. Thus, considering its safety, aesthetic and economic advantages the HNSFRC controllable design strength values are capable of sustaining construction of light-weight, heavy weight, commercial and industrial structures such as Bridge, Building pillars, Sidewalks, Building floors, Drainage pipes, Septic tanks, Concrete Flooring for parking lots, Playgrounds, Airport runways, Taxiways, Maintenance hangars, Access roads, Workshops, Port pavements, Container storage and Handling areas, Bulk storage warehouses, and Military warehouses as high performance concrete

KEYWORDS: Flexural Strength [FS], Split Tensile Strength [STS], Kings-Scheffe's (6,2) Optimization Model, HNSFRC, Mixture Design.

I. INTRODUCTION

Concrete which is the most widely used construction material has been undergoing changes both as a material and due to technological advancement. In an attempt to solve the world's housing deficit partly due to ever increasing rise in the world's population and due to increasing cost of construction materials especially the conventional reinforcement, there have been several research outputs, one being the partial or wholly replacement of the expensive conventional reinforcement with fibres in the reinforced concrete production. By definition, concrete is defined by Oyenuga (2008) as a composite inert material comprising of a binder course (cement), mineral filler or aggregates and water. As stated by Syal and Goel, (2007), concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world. The key role being played by concrete is emphasised by Neville (1990) where he acknowledged that concrete plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. However, Shetty (2006) highlighted some of limitations associated with concrete, especially the unreinforced type. According to him, plain (unreinforced) concrete possesses a very low tensile strength, limited ductility and little resistance to cracking. As a way of finding solution to this situation, there have been continuous search for the upgrading of the concrete properties. In line with this, attempts have been made in the past to improve the tensile properties of concrete members by way of using conventional reinforced steel bars. Despite the fact that both these methods provide tensile strength to the concrete members, it was however observed that they do not

increase the inherent tensile strength of concrete itself. Subsequent upon further researches and recent developments in concrete technology, it has been established that the addition of fibres to concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This type of concrete is known as Fibre Reinforced Concrete [FRC]. Hybrid Fibre Reinforced Concrete [HFRC] is the use of two or more fibres in a single concrete mixture matrix with the aim of improving its overall properties. Hybrid Nylon- Steel Fibre Reinforced Concrete [HNSFRC] is thus concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with the combination of nylon fibre and steel fibre. Special mechanical properties of HNSFRC under investigation in this present work are the flexural strength and the split tensile strength. Flexural strength is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. It is also defined as the maximum bending stress that can be applied to the material before it yields. On the other hand, splitting tensile strength test on concrete cylinder is a method to determine the tensile strength of concrete. It is generally carried out to obtain the tensile strength of concrete, and the stress field in the tests is actually a biaxial stress field with compressive stress three times greater than the tensile stress. The split tensile strength test is an indirect method of testing tensile strength of concrete and is generally greater than direct tensile strength and lower than flexural strength (modulus of rupture).

Combining the two fibres (nylon and steel) in this work with other components can be best achieved through the process of optimization. In general, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated limitations or boundaries placed on the concerned variables. Specifically, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. According to Shacklock (1974), one of the objectives of mix design is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work. Another classic definition by Jackson and Dhir (1996) sees concrete mix design as the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. In line with the above definition, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) appeared to be more complex and time consuming as they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Therefore, optimization of the concrete mixture design remains the fastest method, best option, most convenient and the most efficient way of selecting concrete mix proportions for better efficiency and better performance of concrete when compared with usual empirical methods listed above. An example of optimization model is Scheffe's Model. But Scheffe's Model could only reflect up to four component mixture, with little expansion to five component mixtures. But through modification of the same Scheffe's model, the works of Nwachukwu and others (2022h, 2022i, 2022j, 2023a, 2023b, 2023c, 2023e, 2024a, 2024b, and 2024d) have expanded the use of this Scheffe's model to six component mixtures and still looking forward to publishing works on seven component mixtures. Thus, the modified Scheffe's model in six or seven or higher component mixtures is better described as Kings- Scheffe's Optimization Model. It could be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. Thus, in this present study, Kings-Scheffe's Second Degree Model for six components mixtures (namely, water, cement, fine aggregate, coarse aggregate, nylon fibre and steel fibre) will be in focus.

This present study examines the use of Kings-Scheffe's Second Degree Model for six component mixture in the optimization of the Flexural Strength and Split Tensile Strength of HNSFRC. Although some related works have been done by many researchers, it is worth of note that none of the works has addressed the subject matter substantially. For instance, on NFRC and other related works, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete. Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively. Hassan and others (2022) investigated the Mechanical Properties and Absorption of High-Strength Fiber-Reinforced Concrete (HSFRC) with Sustainable Natural Fibers. On SFRC and other related works, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural

material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate. Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again, Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete. Yew and others (2011) have investigated the strength properties of Hybrid Nylon-Steel fibre-reinforced concrete in comparison to that of polypropylene-steel fibre-reinforced concrete. Recent works on optimization show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For instance, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were based on the use of Scheffe' model in the optimization of compressive strength of Periwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Mbadike and Osadebe (2013) applied Scheffe's (4,2) model to optimize the compressive strength of Laterite Concrete. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation, Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC). Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and Opara (2023) in their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)- Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Nwachukwu and others (2024c)

applied Scheffe’s (5,2) model to evaluate the compressive strength of Plastic Fibre Reinforced Concrete [PLFRC]. Nwachukwu and others (2024d) applied the use of Scheffe’s Third Degree Model to optimize the compressive strength of HPNFRC. Nwachukwu and others (2024e) applied the use of Scheffe’s Third Degree Regression Model to optimize the compressive strength of MSFRC. Nwachukwu and others (2024f) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of NFRC. Again, Nwachukwu and others (2024g) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Nwachukwu and others (2024h) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Finally, Nwachukwu and others (2024i) applied the use of Scheffe’s Second Degree Model to optimize the flexural strength and split tensile strength of SFRC. From the works reviewed so far, there are proofs that no work has been done on the use of Kings- Scheffe’s Second Degree Model to optimize the Flexural Strength and Split Tensile Strength of HNSFRC. Thus, there is urgent need for this present research work.

II. METHODOLOGY

2.1 MATERIALS FOR HNSFRC- FSTS MIXTURES

The constituent materials for FSTS laboratory examination in this present study in line with Kings- Scheffe’s (6,2) model are Water/Cement ratio, Cement, Fine and Coarse Aggregates, Nylon and Steel Fibres . Potable water that is good for concrete is obtained from clean water source and was used in accordance with ASTM C1602/C1602M-22 (2022). The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). Fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. Both fine and coarse aggregates are procured and prepared in accordance with ASTM C33/C33M-18 (2018). The same size and nature of nylon fibre and steel fibre used previously by Nwachukwu and others (2022 d & f) and Nwachukwu and others (2022 b & g) respectively, are the same as the one being used in this present work.

2.2. BACKGROUND OF KINGS-SCHEFFE’S/ SCHEFFE’S (6, 2) MODEL

It is important to note that the difference between the Kings- Scheffe’s model and the original Scheffe’s model is that Kings –Scheffe’s model is the upgraded/ modified/ expanded Scheffe’s model to accommodate from six component mixtures. Thus, many of the technical terms used in the original Scheffe’s model are also used here. Therefore, both models are used here interchangeably here for this six component mixture. As usual, a simplex lattice is described as a structural representation of lines joining the atoms of a mixture where these atoms are constituent components of the mixture. For this present concrete mixture, the six constituent elements are, Water, Cement, Fine Aggregate, Coarse Aggregate, Nylon Fibre and Steel Fibre. According to Obam (2009), mixture components are usually subject to the constraint that the sum of all the components must be equal to 1 as stated in Eqn. (1): $X_1 + X_2 + X_3 + \dots + X_q = 1$; $\Rightarrow \sum_{i=1}^q X_i = 1$

$$(1)$$

where $X_i \geq 0$ and $i = 1, 2, 3 \dots q$, and $q =$ the number of mixtures.

2.2.1. DESIGN POINTS/ NUMBER OF COEFFICIENTS FOR HNSFRC-FSTS KINGS- SCHEFFE’S (6,2) MIXTURES

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen mathematical equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1$; $i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe’s polynomial degee, which in this present study is 2.

For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^4C_2 = 6$ points. Each X_i can take $m+1 = 3$ possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{2}, \frac{1}{2}, 0), (0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2})$. The general formula for evaluating the number of coefficients/terms/ design points required for a given lattice is always given by: k

$$= \frac{(q+m-1)!}{(q-1)! \cdot m!} \text{ Or } {}^{q+m-1}C_m \qquad \qquad \qquad \mathbf{2(a-b)}$$

Where k = number of coefficients/ terms / points, q = number of components = 6 in this study, m = number of degree of polynomial = 2 in this present work. Using either of Eqn. (2), $k_{(6,2)} = 21$

Thus, the 21 possible design points for Kings- Scheffe’s (6, 2) lattice are as stated in Eqn.(3):

$$A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0), A_5 (0,0,0,0,1,0); A_6 (0, 0,0,0, 0, 1); A_{12} (0.67,0.33, 0, 0, 0, 0); A_{13} (0.67, 0, 0.33,0,0,0); A_{14} (0.67, 0, 0, 0.33,0,0); A_{15} (0.67, 0, 0, 0,0.33, 0); A_{16} (0.67, 0, 0, 0, 0, 0.33); A_{23} (0,0.50,0.50, 0,0,0); A_{24} (0, 0.50, 0, 0.50, 0,0); A_{25}, (0, 0.50, 0, 0,0.50, 0); A_{26} (0, 0.50,0,0, 0.50); A_{34} (0.50, 0.50, 0, 0,0,0); A_{35} (0.50, 0,0.50, 0,0,0); A_{36} (0.50,0, 0,0.50, 0, 0); A_{45} (0.50, 0, 0, 0,0.50, 0); A_{46}(0.50,0,0,0,0,0.50);A_{56}(0,0,0.50,0.50,0,0); \tag{3}$$

According to Obam (2009), a Scheffe’s polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \dots X_q$ is given in the form of Eqn.(4)

$$P = b_0 + \sum b_i x_i + \sum b_{ij} x_j + \sum b_{ijk} x_j x_k + \dots + \sum b_{i_1 i_2 \dots i_n} x_{i_1} x_{i_2} \dots x_{i_n} \tag{4}$$

where $(1 \leq i \leq q, 1 \leq i \leq j \leq k \leq q, 1 \leq i_1 \leq i_2 \leq \dots \leq i_n \leq q$ respectively) , b = constant coefficients and P is the response which represents the property under investigation, which in this case is the Flexural Strength (P^F) or Split Tensile Strength (P^S) as the case may be.

As this research work is based on the Scheffe’s (6, 2) simplex, the actual form of Eqn. (4) for six component mixture, degree two (6, 2) has been developed by Nwachukwu and others (2022h) and will be applied subsequently in this work

2.2.2. PSEUDO AND ACTUAL COMPONENTS FOR HNSFRC-FSTS MIXTURE.

In Scheffe’s mixture design, the relationship between the pseudo components and the actual components is given as:

$$Z = A * X \tag{5}$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields: $X = A^{-1} * Z \tag{6}$

2.2.3. FORMULATION OF POLYNOMIAL EQUATION FOR HNSFRC-FSTS KINGS- SCHEFFE’S (6,2) SIMPLEX LATTICE

The polynomial equation by Scheffe (1958), which is known as response is given in Eqn.(4) and for the Kings- Scheffe’s (6,2) simplex lattice, the polynomial equation for six component mixtures has been formulated based on Eqn.(4) by the work of Nwachukwu and others (2022h) as stated under:

$$P = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{16} X_1 X_6 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{26} X_2 X_6 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{36} X_3 X_6 + \beta_{45} X_4 X_5 + \beta_{46} X_4 X_6 + \beta_{56} X_5 X_6 \tag{7}$$

2.2.4. COEFFICIENTS DETERMINATION OF THE HNSFRC-FSTS KINGS-SCHEFFE’S (6, 2) POLYNOMIAL

Based on the work of Nwachukwu and others (2022h), the coefficients of the Kings- Scheffe’s (6, 2) polynomial have been evaluated as stated under. :

$$\begin{aligned} \beta_1 &= P_1; \beta_2 = P_2; \beta_3 = P_3; \beta_4 = P_4; \beta_5 = P_5 \text{ and } \beta_6 = P_6 && \mathbf{8(a-f)} \\ \beta_{12} &= 4P_{12} - 2P_1 - 2P_2; \beta_{13} = 4P_{13} - 2P_1 - 2P_3; \beta_{14} = 4P_{14} - 2P_1 - 2P_4; && \mathbf{9(a-c)} \\ \beta_{15} &= 4P_{15} - 2P_1 - 2P_5; \beta_{16} = 4P_{16} - 2P_1 - 2P_6; \beta_{23} = 4P_{23} - 2P_2 - 2P_3; \beta_{24} = 4P_{24} - 2P_2 - 2P_4; && \mathbf{10(a-d)} \\ \beta_{25} &= 4P_{25} - 2P_2 - 2P_5; \beta_{26} = 4P_{26} - 2P_2 - 2P_6, \beta_{34} = 4P_{34} - 2P_3 - 2P_4; \beta_{35} = 4P_{35} - 2P_3 - 2P_5; && \mathbf{11(a-d)} \\ \beta_{36} &= 4P_{36} - 2P_3 - 2P_6; \beta_{45} = 4P_{45} - 2P_4 - 2P_5, \beta_{46} = 4P_{46} - 2P_4 - 2P_6; \beta_{56} = 4P_{56} - 2P_5 - 2P_6; && \mathbf{12(a-d)} \end{aligned}$$

Where P_i = Response Function (Flexural Strength or Split Tensile Strength) for the pure component, i

2.2.5. HNSFRC-FSTS KINGS- SCHEFFE’S (6, 2) MIXTURE DESIGN MODEL

Substituting Eqns. (8)- (12) into Eqn. (7), we obtain the mixture design model for the HNSFRC –FSTS mixture based on Kings- Scheffe’s (6,2) lattice.

2.2.6. ACTUAL AND PSEUDO MIX PROPORTIONS FOR THE HNSFRC- FSTS KINGS- SCHEFFE’S (6,2) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINT [IETP] AND EXPERIMENTAL CONTROL TEST POINT[ECTP]

A. AT THE HNSFRC- FSTS IETP

Since the requirement of simplex lattice design based on Eqn. (1) criteria makes it impossible to use the conventional mix ratios such as 1:2:4 etc., at a given water/cement ratio for the actual mix ratio., there is need for the transformation of the actual components proportions to meet the above criterion. Based on experience previous knowledge from literature and other related work done on HNSFRC, the following arbitrary prescribed mix ratios are always chosen for the six vertices of Scheffe’s (6,2) lattice when the percentage of Nylon Fibre to Steel Fibre mixture is **50: 50**.

$$A_1 (0.67:1:1.7:2:0.5:0.5); A_2 (0.56:1:1.6:1.8:0.8:0.8); A_3 (0.5:1:1.2:1.7:1:1); A_4 (0.7:1:1:1.8:1.2:1.2); A_5 (0.75:1:1.3:1.2:1.5:1.5), \text{ and } A_6 (0.80:1:1.3:1.2:0.9:0.9) \tag{13}$$

Eqn.(13) mix ratios represent water/cement ratio, cement, fine aggregate, coarse aggregate, nylon fibre and steel fibre respectively. For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures are always chosen:

$$A_1(1:0:0:0:0:0), A_2(0:1:0:0:0:0), A_3(0:0:1:0:0:0), A_4(0:0:0:1:0:0), A_5(0:0:0:0:1:0) \text{ and } A_6(0:0:0:0:0:1) \tag{14}$$

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. Substituting the mix ratios from point A₁ into Eqn. (5) yields:

$$\begin{pmatrix} 0.67 \\ 1.00 \\ 1.70 \\ 2.00 \\ 0.50 \\ 0.50 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

Transforming the R.H.S matrix and solving, we obtain as follows:

A₁₁(1) + A₂₁(0) + A₃₁(0) + A₄₁(0) + A₅₁(0) + A₆₁(0) = 0.67. Thus , A₁₁ = 0.67. Similarly, A₂₁= 1; A₃₁= 1.7; A₄₁= 2; A₅₁= 0.5; A₆₁= 0.5. The same approach is used to obtain the remaining values as shown in Eqn. (16)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} \tag{16}$$

Now considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(16) yields the corresponding actual mix ratios as demonstrated under: For instance, considering point A₁₂ we have: A₁₂ (0.67,0.33, 0, 0, 0, 0). This implies:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.50 & 0.50 & 0.75 & 0.75 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.70 & 1.60 & 1.20 & 1.00 & 1.30 & 1.30 \\ 2.00 & 1.80 & 1.70 & 1.80 & 1.20 & 1.20 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \\ 0.50 & 0.80 & 1.00 & 1.20 & 1.50 & 1.50 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0.63 \\ 1 \\ 1.67 \\ 1.90 \\ 1.60 \\ 1.60 \end{pmatrix} \tag{17}$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$; $Z_4 = 1.90$; $Z_5 = 1.60$ and $Z_6 = 1.60$

The same approach goes for the remaining mid-point mix ratios and twenty-one (21) experimental tests tests (each for Flexural Strength and Split Tensile Strength) are needed to generate the 21 polynomial coefficients based on the corresponding mix ratios as shown in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio for HNSFRC Based on Kings- Scheffe’s (6,2) Lattice For IETP (For Flexural Strength And Split Tensile Strength).

S/N	IETP	PSEUDO COMPONENT						RESPONSE SYMBOL	ACTUAL COMPONENT					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆
1	E ₁	1	0	0	0	0	0	P ₁	0.67	1.00	1.70	2.0	0.5	0.5
2	E ₂	0	1	0	0	0	0	P ₂	0.56	1.00	1.60	1.8	0.8	0.8
3	E ₃	0	0	1	0	0	0	P ₃	0.50	1.00	1.20	1.7	1.0	1.0
4	E ₄	0	0	0	1	0	0	P ₄	0.70	1.00	1.00	1.8	1.2	1.2
5	E ₅	0	0	0	0	1	0	P ₅	0.75	1.00	1.30	1.2	1.5	1.5
6	E ₆	0	0	0	0	0	1	P ₆	0.63	1.00	1.67	1.9	1.6	1.6
7	E ₁₂	0.67	0.33	0	0	0	0	P ₁₂	0.60	1.00	1.63	1.8	0.7	0.7
8	E ₁₃	0.67	0	0.33	0	0	0	P ₁₃	0.61	1.00	1.54	1.9	0.6	0.6
9	E ₁₄	0.67	0	0	0.33	0	0	P ₁₄	0.56	1.00	1.37	1.8	0.8	0.8
10	E ₁₅	0.67	0	0	0	0.33	0	P ₁₅	0.68	1.00	1.47	1.9	0.7	0.7
11	E ₁₆	0.67	0	0	0	0	0.33	P ₁₆	0.69	1.00	1.23	1.8	0.9	0.9
12	E ₂₃	0	0.50	0.50	0	0	0	P ₂₃	0.70	1.00	1.57	1.7	0.8	0.8
13	E ₂₄	0	0.50	0	0.50	0	0	P ₂₄	0.72	1.00	1.43	1.4	1.1	1.1
14	E ₂₅	0	0.50	0	0	0.50	0	P ₂₅	0.55	1.00	1.40	1.7	0.8	0.8
15	E ₂₆	0	0.50	0	0	0	0.50	P ₂₆	0.52	1.00	1.20	1.7	0.9	0.9
16	E ₃₄	0.50	0.50	0	0	0	0	P ₃₄	0.61	1.00	1.67	1.8	0.9	0.9
17	E ₃₅	0.50	0	0.50	0	0	0	P ₃₅	0.66	1.00	1.73	1.8	1.0	1.0
18	E ₃₆	0.50	0	0	0.50	0	0	P ₃₆	0.63	1.00	1.50	1.6	0.7	0.7
19	E ₄₅	0.50	0	0	0	0.50	0	P ₄₅	0.69	1.00	1.40	1.4	0.6	0.6
20	E ₄₆	0.50	0	0	0	0	0.50	P ₄₆	0.57	1.00	1.13	1.7	1.0	1.0
21	E ₅₆	0	0	0.50	0.50	0	0	P ₅₆	0.64	1.00	1.07	1.7	1.1	1.1

B. AT THE HNSFRC-FSTS ECTP

For the purpose of this research, twenty- one (21) different control test (each for Flexural Strength and Split Tensile Strength) were predicted which according to Scheffes, their summation should not be more than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are as shown in Table 2.

Table 2: Actual and Pseudo Component of HNSFRC Based on Scheffe (6,2) Lattice for ECTP (For Flexural Strength And Split Tensile Strength).

S/N	ECTP	PSEUDO COMPONENT						RESPONSE SYMBOL	ACTUAL COMPONENT					
		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆
1	C ₁	0.25	0.25	0.25	0.25	0	0	P ₁	0.61	1	1.38	1.83	0.5	0.50
2	C ₂	0.25	0.25	0.25	0	0.25	0	P ₂	0.62	1	1.45	1.68	0.8	0.8
3	C ₃	0.25	0.25	0	0.25	0.25	0	P ₃	0.67	1	1.40	1.70	1	1
4	C ₄	0.25	0	0.25	0.25	0.25	0	P ₄	0.66	1	1.30	1.68	1.2	1.2
5	C ₅	0	0.25	0.25	0.25	0.25	0	P ₅	0.63	1	1.28	1.63	1.5	1.5
6	C ₆	0.20	0.20	0.20	0.20	0.20	0	P ₆	0.64	1	1.36	1.70	0.65	0.65
7	C ₁₂	0.30	0.30	0.30	0.10	0	0	P ₁₂	0.59	1	1.45	1.83	0.75	0.75
8	C ₁₃	0.30	0.30	0.30	0	0.10	0	P ₁₃	0.59	1	1.48	1.77	0.85	0.85
9	C ₁₄	0.30	0.30	0	0.30	0.10	0	P ₁₄	0.65	1	1.42	1.80	1	1
10	C ₁₅	0.30	0	0.30	0.30	0.10	0	P ₁₅	0.64	1	1.30	1.77	0.9	0.9
11	C ₁₆	0	0.30	0.30	0.30	0.10	0	P ₁₆	0.60	1	1.27	1.71	1	1
12	C ₂₃	0.10	0.30	0.30	0.30	0	0	P ₂₃	0.60	1	1.31	1.79	1.55	1.55
13	C ₂₄	0.30	0.10	0.30	0.30	0	0	P ₂₄	0.62	1	1.33	1.83	1.1	1.1
14	C ₂₅	0.30	0.10	0.30	0.30	0	0	P ₂₅	0.63	1	1.41	1.85	1.25	1.25
15	C ₂₆	0.10	0.20	0.30	0.40	0	0	P ₂₆	0.61	1	1.25	1.79	1.35	1.35
16	C ₃₄	0.30	0.20	0.10	0.40	0	0	P ₃₄	0.64	1	1.35	1.85	0.89	0.89
17	C ₃₅	0.20	0.20	0.10	0.10	0.40	0	P ₃₅	1.40	1	1.04	1.59	1.08	1.08
18	C ₃₆	0.30	0.10	0.30	0.20	0.10	0	P ₃₆	0.62	1	1.36	1.77	0.92	0.92
19	C ₄₅	0.25	0.25	0.15	0.15	0.20	0	P ₄₅	0.61	1	1.51	3.16	0.91	0.91
20	C ₄₆	0.30	0.30	0.20	0.10	0.10	0	P ₄₆	0.68	1	1.56	1.96	0.98	0.98
21	C ₅₆	0.10	0.30	0.30	0.30	0	0	P ₅₆	1.30	1	1.31	1.79	0.95	0.95

2.2.7. MEASUREMENT OF QUANTITIES OF HNSFRC- FSTS MATERIALS

The actual HNSFRC- FSTS components as transformed from Eqn. (17) , Table (1) and (2) were used to measure out the quantities of water/cement ratio (Z₁), cement (Z₂), fine aggregate (Z₃), coarse aggregate (Z₄), nylon fibre (Z₅) and steel fibre (Z₆) in their respective ratios using a weighing balance of 50kg capacity for the eventual Concrete Beam and Cylindrical specimen strength tests at the laboratory.

Mathematically, Measured Quantity, M^Q of HNSFRC Mixture is given by Eqn.(18)

$$M^Q = \frac{X}{T} * Y \tag{18}$$

Where, X = Individual mix ratio at each test point. For example, X = 0.67 for Z₁ at E₁ in Table 1.

T = Sum of mix ratios at each test point = 5.37 at E₁ in Table 1.

And Y = Average weight of Concrete cube/beam/cylinder

For the Flexural Strength concrete beam mould of 15cm*15cm*60cm, Average Y from experience = 30kg

For the Split Tensile Strength Concrete cylinder mould of 15cm*30cm, Average Y from experience =12.5kg
 Samples of measured quantities can be seen from the works of Nwachukwu and others 2024 (a and b).

2.3. METHOD

2.3.1. METHODS FOR HNSFRC FLEXURAL STRENGTH TEST

A. HNSFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR FLEXURAL STRENGTH TEST

In this work, the standard size of specimen (mould) for the Flexural Strength measures 10cm*10cm*50cm as the largest nominal size of the aggregate does not exceed 20mm. The mould is of steel metal with sufficient thickness to prevent

spreading or warping. The mould is constructed with the longer dimension horizontal and in such a manner as to facilitate the removal of the moulded specimen without damage. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. From the twenty one experimental tests, a total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty- one (21) out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (12). Twenty-four (24) hours after moulding, curing commenced. Test specimens are stored in water at a temperature of 24⁰ to 30⁰ for 48 hours before testing. They are tested immediately on removal from the water whilst they are still in a wet condition. After 14 days and 28 days of curing respectively, the specimens were taken out of the curing tank for flexural strength determination.

B. HNSFRC FLEXURAL STRENGTH TEST PROCEDURE/CALCULATION

Flexural strength testing was done in accordance with BS 1881 – part 118 (1983) - Method of determination of Flexural Strength, ASTM C78/C78M-22 (2022) and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the Flexural Strength of each specimen/sample which is expressed as the Modulus of Rupture (MOR) was then calculated to the nearest 0.05 MPa using Eqn.(19)

$$MOR = \frac{PL}{bd^2} \quad (19)$$

where b = measured width in cm of the specimen, d = measured depth in cm of the specimen at the point of failure, where L = Length in cm of the span on which the specimen was supported and P = maximum load in kg applied to the specimen.

2.3.2. METHODS FOR HNSFRC SPLIT TENSILE STRENGTH TEST

A. HNSFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR SPLIT TENSILE STRENGTH TEST

The specimen for the Split Tensile Strength is Concrete Cylindrical specimen measuring diameter 150 mm and length 300 mm. They were cast with plastic fibres and the specimen was loaded for ultimate compressive load under Universal Testing Machine (UTM) for each mix. From the twenty one experimental tests, a total number of 42 mix ratios were to be used to produce 84 prototype concrete cubes. Twenty one (21) out of the 42 mix ratios were as control mix ratios to produce 42 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (12). After 14 and 28 days of curing the specimens were taken out of the curing tank for the Split Tensile Strength determination.

B. HNSFRC SPLIT STRENGTH TEST PROCEDURE/CALCULATION

The cylindrical split tensile test was done using the universal testing machine in accordance with BS EN 12390-6(2009) and ASTM C 496/ C 496 M-11 (2017). Two samples were crushed for each mix ratio and each case, the Split Tensile Strength of each specimen/sample was then calculated using Eqn. (20) .

$$F_t = \frac{2P}{\pi D L} \quad (20)$$

Where, F_t = Split Tensile Strength, MPa , P = maximum applied load (that is Load at failure, N) ; D = diameter of the cylindrical specimen (Dia. Of cylinder, mm); and L = Length of the specimen (Length of cylinder, mm),

III. RESULTS PRESENTATION AND DISCUSSION

3.1. HNSFRC RESPONSES (FLEXURAL STRENGTH) FOR THE INITIAL EXPERIMENTAL TEST

The results of the Flexural Strength (responses) from the IETP based on Eqn. (19) are shown in Table 3

Table 3: HNSFRC Flexural Strength (Response) For The IETP Based on Eqn.(19)

S/N	IE TP	REPLICATE	RESPONSE SYMBOL	RESPONSE P _i , MPa		ΣP _i		AVERAGE RESPONSE P, MPa	
				14 th day Results	28 th day Results	14 th day Results	28 th day Results	14 th day Results	28 th day Results
1	E ₁	HNSFRC/FS/ E ₁ A HNSFRC/FS/ E ₁ B	P ₁	5.00 5.03	5.35 5.40	10.03	10.75	5.02	5.38
2	E ₂	HNSFRC/FS/ E ₂ A HNSFRC/FS/ E ₂ B	P ₂	5.45 5.49	6.98 7.00	10.94	13.98	5.47	6.99
3	E ₃	HNSFRC/FS/ E ₃ A HNSFRC/FS/ E ₃ B	P ₃	5.88 5.83	7.45 7.48	11.71	14.93	5.86	7.47
4	E ₄	HNSFRC/ FS/E ₄ A HNSFRC/FS/ E ₄ B	P ₄	6.32 6.34	8.43 8.48	12.66	16.91	6.33	8.46
5	E ₅	HNSFRC/FS/ E ₅ A HNSFRC/FS/ E ₅ B	P ₅	4.89 4.92	6.75 6.79	9.81	13.54	4.91	6.77
6	E ₆	HNSFRC/FS/ E ₆ A HNSFRC/FS/ E ₆ B	P ₆	6.77 6.73	10.41 10.37	13.50	20.78	6.75	10.39
7	E ₁₂	HNSFRC/FS/ E ₁₂ A HNSFRC/FS/ E ₁₂ B	P ₁₂	5.23 5.34	7.56 7.60	10.57	15.16	5.29	7.58
8	E ₁₃	HNSFRC/FS/ E ₁₃ A HNSFRC/FS/ E ₁₃ B	P ₁₃	5.68 5.75	9.43 9.47	11.43	18.90	5.72	9.45
9	E ₁₄	HNSFRC/FS/ E ₁₄ A HNSFRC/FS/ E ₁₄ B	P ₁₄	6.12 6.21	9.68 9.70	12.33	18.38	6.17	9.69
10	E ₁₅	HNSFRC/ FS/E ₁₅ A HNSFRC/FS/ E ₁₅ B	P ₁₅	5.69 5.72	10.34 10.35	11.41	20.69	5.71	10.35
11	E ₁₆	HNSFRC/FS/ E ₁₆ A HNSFRC/FS/ E ₁₆ B	P ₁₆	6.50 6.54	9.47 9.49	13.04	18.96	6.52	9.48
12	E ₂₃	HNSFRC/FS/ E ₂₃ A HNSFRC/FS/ E ₂₃ B	P ₂₃	5.45 5.48	8.38 8.41	10.93	16.79	5.47	8.40
13	E ₂₄	HNSFRC/FS/ E ₂₄ A HNSFRC/FS/ E ₂₄ B	P ₂₄	5.21 5.28	6.48 6.50	10.49	12.98	5.25	6.49
14	E ₂₅	HNSFRC/ FS/E ₂₅ A HNSFRC/FS/ E ₂₅ B	P ₂₅	5.29 5.36	6.75 6.76	10.65	13.51	5.33	6.76

15	E ₂₆	HNSFRC/FS/ E ₂₆ A HNSFRC/FS/ E ₂₆ B	P ₂₆	6.22 6.33	7.28 7.32	12.55	14.66	6.28	7.33
16	E ₃₄	HNSFRC/ FS/E ₃₄ A HNSFRC/FS/ E ₃₄ B	P ₃₄	4.92 4.96	5.26 5.30	9.88	10.56	4.94	5.28
17	E ₃₅	HNSFRC/ FS/E ₃₅ A HNSFRC/FS/ E ₃₅ B	P ₃₅	5.42 5.47	6.47 6.53	10.89	13.00	5.45	6.50
18	E ₃₆	HNSFRC/FS/ E ₃₆ A HNSFRC/FS/ E ₃₆ B	P ₃₆	5.42 5.39	7.46 7.49	10.81	14.95	5.41	7.48
19	E ₄₅	HNSFRC/ FS/E ₄₅ A HNSFRC/FS/ E ₄₅ B	P ₄₅	5.37 5.42	8.42 8.38	10.79	16.80	5.40	8.40
20	E ₄₆	HNSFRC/ FS/E ₄₆ A HNSFRC/ FS/E ₄₆ B	P ₄₆	5.13 5.18	9.42 9.49	10.31	18.91	5.16	9.46
21	E ₅₆	HNSFRC/ FS/E ₅₆ A HNSFRC/ FS/E ₅₆ B	P ₅₆	5.23 5.36	6.89 6.85	10.59	13.74	5.30	6.87

3.2. HNSFRC RESPONSES (SPLIT TENSILE STRENGTH) FOR THE IETP.

The results of the Split Tensile Strength (response) for the IETP based on Eqn. (20) are shown in Table 4

Table 4: HNSFRC Split Tensile Strength (Response) For The IETP Based on Eqn.(20)

S/N	IE TP	REPLICATE	RESPONSE SYMBOL	RESPONSE P _i , MPa		ΣP _i		AVERAGE RESPONSE,P MPa	
				14 th day Results	28 th day Results	14 th day Results	28 th day Results	14 th day Results	28 th day Results
1	E ₁	HNSFRC/STS/ E ₁ A HNSFRC/STS/ E ₁ B	P ₁	4.00 4.07	4.53 4.56	8.07	9.09	4.04	4.55
2	E ₂	HNSFRC/STS/ E ₂ A HNSFRC/STS/ E ₂ B	P ₂	3.76 3.79	5.65 5.71	7.55	11.36	3.78	5.68
3	E ₃	HNSFRC/STS/ E ₃ A HNSFRC/STS/ E ₃ B	P ₃	3.87 3.91	4.74 4.79	7.78	9.53	3.89	4.77
4	E ₄	HNSFRC/ STS/E ₄ A HNSFRC/STS/ E ₄ B	P ₄	3.71 3.72	6.54 6.56	7.43	13.10	3.72	6.55
5	E ₅	HNSFRC/STS/ E ₅ A HNSFRC/STS/ E ₅ B	P ₅	3.93 3.94	4.54 4.48	7.87	9.02	3.94	4.51
6	E ₆	HNSFRC/STS/ E ₆ A HNSFRC/STS/ E ₆ B	P ₆	4.48 4.44	6.82 6.78	8.92	13.60	4.46	6.80
7	E ₁₂	HNSFRC/STS/ E ₁₂ A HNSFRC/STS/ E ₁₂ B	P ₁₂	3.84 3.85	5.43 5.48	7.69	10.91	3.85	5.46

8	E ₁₃	HNSFRC/STS/ E ₁₃ A HNSFRC/STS/ E ₁₃ B	P ₁₃	4.00 4.01	5.43 5.52	8.01	10.95	4.01	5.48
9	E ₁₄	HNSFRC/STS/ E ₁₄ A HNSFRC/STS/ E ₁₄ B	P ₁₄	3.75 3.78	4.76 4.78	7.53	9.54	3.77	4.77
10	E ₁₅	HNSFRC/ STS/E ₁₅ A HNSFRC/STS/ E ₁₅ B	P ₁₅	3.99 3.88	4.65 4.57	7.87	9.22	3.94	4.61
11	E ₁₆	HNSFRC/STS/ E ₁₆ A HNSFRC/STS/ E ₁₆ B	P ₁₆	3.53 3.61	5.87 5.90	7.14	11.77	3.57	5.89
12	E ₂₃	HNSFRC/STS/ E ₂₃ A HNSFRC/STS/ E ₂₃ B	P ₂₃	3.86 3.82	5.43 5.46	7.68	10.89	3.84	5.45
13	E ₂₄	HNSFRC/STS/ E ₂₄ A HNSFRC/STS/ E ₂₄ B	P ₂₄	3.86 3.83	4.54 4.58	7.69	9.12	3.85	4.46
14	E ₂₅	HNSFRC/ STS/E ₂₅ A HNSFRC/STS/ E ₂₅ B	P ₂₅	4.13 4.21	5.47 5.58	8.34	11.05	4.17	5.53
15	E ₂₆	HNSFRC/STS/ E ₂₆ A HNSFRC/STS/ E ₂₆ B	P ₂₆	4.19 4.17	5.23 5.31	8.34	10.54	4.18	5.27
16	E ₃₄	HNSFRC/ STS/E ₃₄ A HNSFRC/STS/ E ₃₄ B	P ₃₄	3.52 3.48	4.24 4.20	7.00	8.44	3.50	4.22
17	E ₃₅	HNSFRC/ STS/E ₃₅ A HNSFRC/STS/ E ₃₅ B	P ₃₅	3.85 3.84	4.76 4.87	7.69	9.63	3.85	4.82
18	E ₃₆	HNSFRC/STS/ E ₃₆ A HNSFRC/STS/ E ₃₆ B	P ₃₆	3.74 3.68	5.64 5.78	7.42	11.38	3.71	5.69
19	E ₄₅	HNSFRC/ STS/E ₄₅ A HNSFRC/STS/ E ₄₅ B	P ₄₅	3.56 3.58	6.00 6.03	7.12	12.03	3.56	6.02
20	E ₄₆	HNSFRC/ STS/E ₄₆ A HNSFRC/ STS/E ₄₆ B	P ₄₆	4.23 4.28	5.84 5.86	8.51	11.70	4.26	5.85
21	E ₅₆	HNSFRC/ STS/E ₅₆ A HNSFRC/ STS/E ₅₆ B	P ₅₆	4.40 4.41	6.23 6.34	8.81	12.57	4.41	6.29

3.3. HNSFRC RESPONSES (FLEXURAL STRENGTH) FOR THE ECTP

The response (Flexural strength) from the ECTP is shown in Table 5.

Table 5: HNSFRC Response (Flexural strength) Of The ECTP.

S/ N	EC TP	REPLICATE	RESPONSE MPa		L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	AVERAGE RESPONSE, MPa			
			14 th day Results	28 th day Results							14 th day Results	28 th day Results		
1	C ₁	HNSFRC/FS/ C ₁ A	5.03	5.31	0.61	1	1.38	1.83	0.5	0.50	5.02	5.35	10.42	
		HNSFRC/FS/ C ₁ B	5.00	5.38										
2	C ₂	HNSFRC/FS/ C ₂ A	5.40	6.91	0.62	1	1.45	1.68	0.8	0.8	5.41	6.97	9.04	
		HNSFRC/FS/ C ₂ B	5.41	7.02										
3	C ₃	HNSFRC/FS/ C ₃ A	5.81	7.41	0.67	1	1.40	1.70	1	1	5.80	7.42	7.33	
		HNSFRC/FS/ C ₃ B	5.79	7.42										
4	C ₄	HNSFRC/FS/ C ₄ A	6.29	8.39	0.66	1	1.30	1.68	1.2	1.2	6.30	8.40	7.89	
		HNSFRC/FS/ C ₄ B	6.31	8.41										
5	C ₅	HNSFRC/FS/ C ₅ A	4.81	6.71	0.63	1	1.28	1.63	1.5	1.5	4.83	6.73	12.81	
		HNSFRC/FS/ C ₅ B	4.85	6.74										
6	C ₆	HNSFRC/FS/ C ₆ A	6.71	10.28	0.64	1	1.36	1.70	0.65	0.65	6.71	10.30	10.77	
		HNSFRC/FS/ C ₆ B	6.70	10.32										
7	C ₁₁	HNSFRC/FS/ C ₁₁ A	5.19	7.51	0.59	1	1.45	1.83	0.75	0.75	5.25	7.54	7.6	
		HNSFRC/FS/ C ₁₁ B	5.30	7.57										
8	C ₁₁	HNSFRC/FS/ C ₁₁ A	5.61	9.38	0.59	1	1.48	1.77	0.85	0.85	5.66	9.40	8.1	
		HNSFRC/FS/ C ₁₁ B	5.71	9.42										
9	C ₁₄	HNSFRC/FS/ C ₁₄ A	6.24	9.61	0.65	1	1.42	1.80	1	1	6.27	9.66	7.05	
		HNSFRC/FS/ C ₁₄ B	6.29	9.70										
10	C ₁₅	HNSFRC/FS/ C ₁₅ A	5.61	10.28	0.64	1	1.30	1.77	0.9	0.9	5.67	10.30	7.25	
		HNSFRC/FS/ C ₁₅ B	5.73	10.31										
11	C ₁₆	HNSFRC/FS/ C ₁₆ A	6.64	9.41	0.60	1	1.27	1.71	1	1	6.68	9.42	8.04	
		HNSFRC/FS/ C ₁₆ B	6.71	9.42										
12	C ₂₁	HNSFRC/FS/ C ₂₁ A	5.39	8.31	0.60	1	1.31	1.79	1.55	1.55	5.40	8.32	7.96	
		HNSFRC/FS/ C ₂₁ B	5.40	8.32										
13	C ₂₄	HNSFRC/FS/ C ₂₄ A	5.19	6.41	0.62	1	1.33	1.83	1.1	1.1	5.20	6.42	8.14	
		HNSFRC/FS/ C ₂₄ B	5.20	6.42										
14	C ₂₅	HNSFRC/FS/ C ₂₅ A	5.23	6.71	0.63	1	1.41	1.85	1.25	1.25	5.29	6.71	10.54	
		HNSFRC/FS/ C ₂₅ B	5.34	6.70										
15	C ₂₆	HNSFRC/FS/ C ₂₆ A	6.20	7.22	0.61	1	1.25	1.79	1.35	1.35	6.26	7.25	11.02	
		HNSFRC/FS/ C ₂₆ B	6.31	7.27										

16	C ₁₄	HNSFRC/FS/C ₁₄ A HNSFRC/FS/ C ₁₄ B	4.86 4.78	5.21 5.28	0.64	1	1.35	1.85	0.89	0.89	4.82	5.25
17	C ₁₅	HNSFRC/FS/C ₁₅ A HNSFRC/FS/ C ₁₅ B	5.41 5.41	6.42 6.47	1.40	1	1.04	1.59	1.08	1.08	5.41	6.45
18	C ₁₆	HNSFRC/FS/ C ₁₆ A HNSFRC/FS/ C ₁₆ B	5.39 5.31	7.42 7.41	0.62	1	1.36	1.77	0.92	0.92	5.40	7.42
19	C ₂₀	HNSFRC/FS/C ₂₀ A HNSFRC/FS/ C ₂₀ B	5.33 5.39	8.38 8.31	0.61	1	1.51	3.16	0.91	0.91	5.36	8.35
20	C ₂₆	HNSFRC/FS/C ₂₆ A HNSFRC/FS/C ₂₆ B	5.17 5.21	9.32 9.37	0.68	1	1.56	1.96	0.98	0.98	5.19	9.35
21	C ₂₈	HNSFRC/FS/C ₂₈ A HNSFRC/FS/C ₂₈ B	5.22 5.31	6.82 6.84	1.30	1	1.31	1.79	0.95	0.95	5.27	6.83

3.4. HNSFRC RESPONSES (SPLIT TENSILE STRENGTH) FOR THE ECTP.

The response (Split Tensile Strength) from the ECTP is shown in Table 6.

Table 6: HNSFRC Response (Split Tensile Strength) from ECTP.

S/N	EC TP	REPLICATE	RESPONSE MPa		Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆	AVERAGE RESPONSE, MPa		
			14 th day Results	28 th day Results							14 th day Results	28 th day Results	
1	C ₁	HNSFRC/STS/C ₁ A HNSFRC/STS/ C ₁ B	4.08 4.09	4.49 4.51	0.61	1	1.38	1.83	0.5	0.50	4.09	4.50	10.42
2	C ₂	HNSFRC/STS/C ₂ A HNSFRC/STS/ C ₂ B	3.72 3.65	3.58 3.65	0.62	1	1.45	1.68	0.8	0.8	3.69	3.62	9.04
3	C ₃	HNSFRC/STS/ C ₃ A HNSFRC/STS/ C ₃ B	3.68 3.73	4.71 4.72	0.67	1	1.40	1.70	1	1	3.71	4.72	7.33
4	C ₄	HNSFRC/STS/C ₄ A HNSFRC/STS/ C ₄ B	3.79 3.65	6.51 6.53	0.68	1	1.30	1.68	1.2	1.2	3.72	6.52	7.89
5	C ₅	HNSFRC/STS/C ₅ A HNSFRC/STS/ C ₅ B	3.92 3.89	4.48 4.42	0.63	1	1.28	1.63	1.5	1.5	3.91	4.45	12.81
6	C ₆	HNSFRC/STS/ C ₆ A HNSFRC/STS/ C ₆ B	4.40 4.47	6.76 6.72	0.64	1	1.36	1.70	0.65	0.65	4.44	6.74	10.77
7	C ₁₁	HNSFRC/STS/C ₁₁ A HNSFRC/STS/C ₁₁ B	3.78 3.81	3.39 3.42	0.59	1	1.45	1.83	0.75	0.75	3.80	3.41	7.6
8	C ₁₁	HNSFRC/STS/C ₁₁ A HNSFRC/STS/C ₁₁ B	4.03 4.05	5.41 5.45	0.59	1	1.48	1.77	0.85	0.85	4.04	5.41	8.1
9	C ₁₁	HNSFRC/STS/C ₁₁ A HNSFRC/STS/C ₁₁ B	3.69 3.74	4.72 4.67	0.65	1	1.42	1.80	1	1	3.72	4.70	7.05
10	C ₁₁	HNSFRC/ STS/C ₁₁ A HNSFRC/STS/ C ₁₁ B	3.86 3.82	4.61 4.63	0.64	1	1.30	1.77	0.9	0.9	3.84	4.62	7.25
11	C ₁₆	HNSFRC/STS/ C ₁₆ A HNSFRC/STS/ C ₁₆ B	3.45 3.52	5.92 5.90	0.60	1	1.27	1.71	1	1	3.49	5.91	8.04
12	C ₂₁	HNSFRC/STS/ C ₂₁ A HNSFRC/STS/ C ₂₁ B	3.76 3.83	5.48 5.43	0.60	1	1.31	1.79	1.55	1.55	3.80	5.46	7.96
13	C ₂₁	HNSFRC/STS/ C ₂₁ A HNSFRC/STS/ C ₂₁ B	3.82 3.74	4.53 4.52	0.62	1	1.33	1.83	1.1	1.1	3.78	4.53	8.14
14	C ₂₄	HNSFRC/ STS/C ₂₄ A HNSFRC/STS/ C ₂₄ B	4.11 4.15	5.43 5.45	0.63	1	1.41	1.85	1.25	1.25	4.13	5.44	10.54
15	C ₂₆	HNSFRC/STS/ C ₂₆ A HNSFRC/STS/ C ₂₆ B	4.13 4.12	5.39 5.42	0.61	1	1.25	1.79	1.35	1.35	4.13		11.02
16	C ₂₄	HNSFRC/ STS/C ₂₄ A HNSFRC/STS/ C ₂₄ B	3.43 3.43	4.21 4.19	0.64	1	1.35	1.85	0.89	0.89	3.43	4.20	
17	C ₂₄	HNSFRC/ STS/C ₂₄ A HNSFRC/STS/ C ₂₄ B	3.82 3.79	4.72 4.73	1.40	1	1.04	1.59	1.08	1.08	3.81	4.73	

18	C ₃₆	HNSFRC/STS/C ₃₆ A HNSFRC/STS/C ₃₆ B	3.72 3.73	5.64 5.68	0.62	1	1.36	1.77	0.92	0.92	3.73	5.66		
19	C ₄₅	HNSFRC/STS/C ₄₅ A HNSFRC/STS/C ₄₅ B	3.31 3.52	6.03 6.02	0.61	1	1.51	3.16	0.91	0.91	3.32	6.03		
20	C ₄₅	HNSFRC/STS/C ₄₅ A HNSFRC/STS/C ₄₅ B	4.32 4.34	5.78 5.79	0.68	1	1.56	1.96	0.98	0.98	4.33	5.79		
21	C ₃₆	HNSFRC/STS/C ₃₆ A HNSFRC/STS/C ₃₆ B	4.42 4.47	6.43 6.48	1.30	1	1.31	1.79	0.95	0.95	4.45	6.46		

3.5. KINGS-SCHEFFE’ S (6,2) POLYNOMIAL MODEL FOR THE HNSFRC RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT).

A. HNSFRC FLEXURAL STRENGHT

By substituting the values of the flexural strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients (β₁ , β₂ ... β₃₄ ,β₃₅... B₅₆) of the Kings-Scheffe’s second degree polynomial for HNSFRC Substituting the values of these coefficients into Eqn. (7) yields the polynomial model for the optimization of the flexural strength of HNSFRC (at 14th day or 28th day) based on Kings-Scheffe’s (6,2) lattice as given in Eqn.(21)

$$P^F = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_6X_6 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{16}X_1X_6 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{26}X_2X_6 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{36}X_3X_6 + \beta_{45}X_4X_5 + \beta_{46}X_4X_6 + \beta_{56}X_5X_6$$

(21)

B. HNSFRC SPLIT TENSILE STRENGHT

By substituting the values of the split tensile strengths (responses) from Table 4 into Eqns.(8) through (10), we obtain the coefficients (β₁ , β₂ ... β₃₄ ,β₃₅... B₅₆) of the Scheffe’s second degree polynomial for HNSFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the split tensile strength of HNSFRC (at 14th day or 28th day) based on Scheffe’s (6,2) lattice as given in Eqn.(22)

$$P^S = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_6X_6 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{16}X_1X_6 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{26}X_2X_6 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{36}X_3X_6 + \beta_{45}X_4X_5 + \beta_{46}X_4X_6 + \beta_{56}X_5X_6$$

(22)

3.6. KINGS- SCHEFFE’S (6,2) MODEL RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) FOR HNSFRC AT ECTP.

A. HNSFRC FLEXURAL STRENGHT MODEL RESPONSE AT ECTP.

By substituting the pseudo mix ratio of points C₁, C₂, C₃, C₄, C₅, ... C₅₆ of Table 5 into Eqn.(21), we obtain the Kings-Scheffe’s second degree model responses (flexural strength) for the ECTP of HNSFRC

B. HNSFRC SPLIT TENSILE STRENGHT MODEL RESPONSE AT ECTP.

By substituting the pseudo mix ratio of points C₁, C₂, C₃, C₄, C₅, ... C₅₆ of Table 6 into Eqn.(22), we obtain the second degree model responses (split tensile strength) for the ECTP of HNSFRC

3.7. VALIDATION AND TEST OF ADEQUACY OF HNSFRC MODEL RESULTS (FOR FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) USING STUDENT’S – T -TEST

Here, the test of adequacy is performed in order to know if there is a good correlation between the flexural and split tensile strengths results (lab responses) given in Tables 5 and 6 and model responses from the control points based on Eqns.(21 and 22). By using the Student’s – T – test as the means of validation, the result shows that there are no significant differences between the experimental results and model responses. The steps involved in using the Student’s – T - test have been explained by Nwachukwu and others (2022 c). Therefore, the models are adequate for predicting the flexural and split tensile strengths of HNSFRC based on Kings-Scheffe’s (6,2) simplex lattice.

3.8. RESULTS DISCUSSION

From the results, the maximum flexural strengths of HNSFRC based on Kings-Scheffe's (6, 2) lattice are **10.39 MPa** and **6.75 MPa** respectively for 28th and 14th day results. Similarly the maximum split tensile strengths of HNSFRC based on Scheffe's (6,2) lattice are **6.80 MPa** and **4.46MPa** respectively for 28th and 14th day results. The corresponding optimum mix ratio is **0.63:1.00: 1.67:1.9:1.6:1.6** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Nylon Fibre and Steel Fibre respectively. The minimum flexural strength and split tensile strength are **5.28 MPa**, **4.94 MPa**, **4.22MPa** and **3.50MPa** respectively for the 28th day and 14th day results. The minimum values correspond to the mix ratio of **0.61:1.00:1.67:1.8:0.9:0.9** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Nylon Fibre and Steel Fibre respectively. Thus, the Scheffe's model can be used to determine the HNSFRC flexural and split tensile strength of all points (1 - 56) in the simplex based on Kings- Scheffe's Second Degree Model for six component mixture.

IV. CONCLUSION

So far in this work, Kings-Scheffe's Second Degree Polynomial (6, 2) has been presented and used to formulate a model for predicting the flexural and split tensile strengths of HNSFRC. In the first instance, the Scheffe's model was used to predict the mix ratio for predicting both flexural and split tensile strengths of HNSFRC. Through the use of Kings-Scheffe's (6,2) simplex model, the values of both strengths were determined at all 21 points (1 - 56). The results of the student's t-test validated the design strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable strengths predicted by the model based on Scheffe's (6,2) model are as stated in the results discussion session. Thus, with the Kings-Scheffe's (6,2) model, any desired strength, given any mix ratio can be easily predicted and evaluated and vice versa. Thus, the application of this Scheffe's optimization model has reduced the problem of having to go through vigorous, time-consuming and laborious empirical mixture design procedures in order to obtain the desired strengths of HNSFRC mixture.

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